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# COUETTE PROBLEM FOR A RAREFIED GAS IN A CHANNEL

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*A study is made of the influence of the accommodation coefficients of tangential momentum of molecules on the lower and upper plates on the behavior of the gas between moving plates with arbitrary mirror-diffuse boundary conditions. The solution is constructed in a wide range of Knudsen numbers. Expressions for the mass and heat fluxes, the friction force, and the mass velocity are obtained.*

**Introduction.** In describing the motion of a gas in channels [1–6], one considers purely diffuse boundary conditions, as a rule. It turns out that the efficient technique of analytical solution that has been developed for half-spatial problems [7] cannot be used directly in this case. At the same time, the influence of the properties of the surface of the channel behavior of the gas is of great interest. An attempt at obtaining the analytical solution for nearly mirror boundary conditions has been made in [6]. An extensive amount of literature (see, e.g., [8–10] and the references therein) is devoted to the gas motion in a channel at present.

In this work, we have obtained the solution of the Couette problem with arbitrary mirror-diffuse conditions on channel walls moving in opposite directions with the same velocities.

**Formulation of the Problem.** Let there be a plane channel ($|x| < d$) whose walls move in their planes in the opposite directions with velocities $U$ and $-U$. We introduce a Cartesian coordinate system with its center at the center of the channel; the $x$ axis is perpendicular to the channel walls, and the $z$ axis coincides with the direction of their motion. We will assume that the motion is stationary in character. We consider the case where the velocity of motion of the channel walls is much lower than the velocity of sound in the gas. Therefore, the problem can be linearized. The velocity-distribution function $f$ of the gas molecules will be sought in the form $f = f_0(1 + h)$. We use a linearized BGK (Bhatnagar, Gross, and Krook) equation in dimensionless variables:

$$C_x \frac{\partial h}{\partial x} + h(x, \mathbf{C}) = 2C_z U_z(x) , \quad U_z(x) = \pi^{-3/2} \int \exp(-C'^2)\, C_z' h(x, \mathbf{C}')\, d^3C' . \tag{1}$$

We consider mirror-diffuse boundary conditions on the channel surface with tangential-momentum accommodation coefficients (coefficients of specular reflection) $q_1$ and $q_2$ ($0 \le q_j \le 1$ and $j = 1, 2$):

$$h(-d, \mathbf{C}) = (1 - q_1)\, h(-d, \mathbf{C} + 2\mathbf{n}_1\mathbf{C}) - 2UC_z q_1 , \quad C_x > 0 ;$$
$$h(d, \mathbf{C}) = (1 - q_1)\, h(d, \mathbf{C} + 2\mathbf{n}_2\mathbf{C}) + 2UC_z q_1 , \quad C_x < 0 . \tag{2}$$

It follows from Eq. (1) and boundary conditions (2) that the function $h$ can be sought as $h = C_z\psi(x, \mu)$, $\mu = C_x$. Problem (1) and (2) can be transformed as follows:

$$\mu \frac{\partial \psi}{\partial x} + \psi(x, \mu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-\mu'^2)\, \psi(x, \mu')\, d\mu' , \tag{3}$$

$$\psi(-d, \mu) = (1 - q_1)\, \psi(-d, -\mu) - 2Uq_1 , \quad \mu > 0 ; \quad \psi(d, \mu) = (1 - q_2)\, \psi(d, -\mu) + 2Uq_2 , \quad \mu < 0 . \tag{4}$$

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We will follow the method presented in [10]. Representing the function $\psi$ as the sum

$$\psi = \psi_0(x, \mu) + \psi_c(x, \mu) , \tag{5}$$

where the function $\psi_0(x, \mu) = a_0 + a_1(x - \mu)$ is the solution of Eq. (3), we rewrite boundary conditions (4) in the form

$$\begin{aligned} \psi_c(-d, \mu) &= (1 - q_1)\, \psi_c(-d, -\mu) + a_1(2 - q_1)\, \mu - q_1(a_0 - a_1 d + 2U) , \quad \mu > 0 ; \\ \psi_c(d, \mu) &= (1 - q_2)\, \psi_c(d, -\mu) + a_1(2 - q_2)\, \mu - q_2(a_0 + a_1 d - 2U) , \quad \mu < 0. \end{aligned} \tag{6}$$

or, introducing the notation

$$\psi_c(-d, \mu) = C_1(\mu) \equiv C_1 , \quad \mu < 0 ; \quad \psi_c(d, \mu) = C_2(\mu) \equiv C_2 , \quad \mu > 0 ,$$

we obtain that boundary conditions (6) have been determined on the entire number axis

$$\psi_c(\pm d, \mu) = M_\pm(\mu) , \quad -\infty < \mu < \infty . \tag{7}$$

Here we have

$$M_-(\mu) = H_+(\mu)\, \varphi_1(\mu) + H_-(-\mu)\, C_1 , \quad M_+(\mu) = H_+(-\mu)\, \varphi_2(\mu) + H_+(\mu)\, C_2 ; \tag{8}$$

$$\varphi_1(\mu) = (1 - q_1)\, C_1 - q_1(a_0 - a_1 d + 2U) + a_1(2 - q_1)\, \mu ;$$

$$\varphi_2(\mu) = (1 - q_2)\, C_2 - q_2(a_0 + a_1 d - 2U) + a_1(2 - q_2)\, \mu .$$

Next we will solve Eq. (3) with boundary conditions (7).

**Solution of the Boundary-Value Problem.** Separation of variables

$$\psi_\eta(x, \mu) = \exp\left(-\frac{x}{\eta}\right) \Phi(\eta, \mu) , \quad \eta \in C$$

immediately reduces Eq. (3) to the characteristic equation:

$$(\eta - \mu)\, \Phi(\eta, \mu) = \frac{\eta}{\sqrt{\pi}} , \quad \int_{-\infty}^{\infty} \exp(-\mu^2)\, \Phi(\eta, \mu)\, d\mu = 1 .$$

When $-\infty < \eta < \infty$ the solution of this equation will be taken in the space of generalized functions [11]:

$$\Phi(\eta, \mu) = \frac{1}{\sqrt{\pi}}\, \eta P \frac{1}{\eta - \mu} + \exp(\eta^2)\, \lambda(\eta - \mu) , \quad \lambda(z) = 1 + \frac{z}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-v^2)\, dv}{v - z} .$$

We compose the general solution of Eq. (3) from the eigenfunctions $\Phi$ of the characteristic equation

$$\psi_c(x, \mu) = \int_{-\infty}^{\infty} \exp\left(-\frac{x}{\eta}\right) \Phi(\eta, \mu)\, a(\eta)\, d\eta , \tag{9}$$

where $a(\eta)$ is the unknown function called the continuous-spectrum coefficient.

Substituting the expansion (9) into boundary conditions (7), we obtain two singular integral equations [12] with a Cauchy kernel on the entire number axis:

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp\left(\pm\frac{d}{\eta}\right)\frac{\eta a(\eta)\,d\eta}{\eta-\mu}+\exp\left(\mu^2\pm\frac{d}{\mu}\right)\lambda(\mu)\,a(\mu)=M_{\pm}(\mu)\,. \tag{10}$$

We introduce two auxiliary functions

$$N(z,\pm d)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp\left(\mp\frac{d}{\eta}\right)\frac{\eta a(\eta)\,d\eta}{\eta-\mu}\,, \tag{11}$$

for which the Sohotskii formulas

$$N^{+}(\mu,\mp d)-N^{-}(\mu,\mp d)=2\sqrt{\pi}\,i\mu\exp\left(\pm\frac{d}{\mu}\right)a(\mu) \tag{12}$$

hold on the entire real axis. Using the boundary values of the auxiliary functions (11) and the dispersion function $\lambda(z)$, we reduce Eqs. (10) to the boundary-value problems of determination of the analytical function from its jump on the real axis:

$$N^{+}(\mu,\mp d)\,\lambda^{+}(\mu)-N^{-}(\mu,\mp d)\,\lambda^{-}(\mu)=2\sqrt{\pi}\,i\mu\exp(-\mu^2)\,M_{\mp}(\mu)\,.$$

The solutions of these problems are expressed by integrals of the Cauchy type:

$$N(z,\mp d)=\frac{F(z,\mp d)}{\lambda(z)}\,,\quad F(z,\mp d)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\tau\exp(-\tau^2)}{\tau-z}M_{\mp}(\tau)\,d\tau\,. \tag{13}$$

It is necessary that the expansion coefficients of these functions in the vicinity of the point $z=\infty$ for $z^k$ ($k=0,1$) are equal to zero. Then, taking the functions of (13) as the auxiliary functions (11), we obtain the following system of equations:

$$\int_{-\infty}^{\infty}\exp(-t^2)\,t^k\,[C_1H_{+}(-t)+\varphi_1(t)\,H_{+}(t)]\,dt=0\,,$$

$$\int_{-\infty}^{\infty}\exp(-t^2)\,t^k\,[C_2H_{+}(t)+\varphi_2(t)\,H_{+}(-t)]\,dt=0\,,\quad k=1,2\,.$$

Substitution of the function of (8) into these equations yields a system of linear equations from which we find

$$a_0=-2U\frac{1+\alpha_0q_1q_2}{Q(q_1,q_2)}(q_1-q_2)\,,\quad a_1=\frac{4Uq_1q_2}{\sqrt{\pi}\,Q(q_1,q_2)}\,,\quad \alpha_0=\frac{4-\pi}{4\pi}=0.068310\,, \tag{14}$$

$$C_1=-4\alpha_0U\frac{(2-q_1)\,q_1q_2}{Q(q_1,q_2)}\,,\quad C_2=4\alpha_0U\frac{(2-q_2)\,q_1q_2}{Q(q_1,q_2)}\,,\quad \alpha_0=\frac{4-\pi}{4\pi}=0.068310\,, \tag{15}$$

$$Q(q_1,q_2)=\left(\frac{2d}{\sqrt{\pi}}-1+4\alpha_0\right)q_1q_2+(1-\alpha_0q_1q_2)(q_1+q_2)\,.$$

Using equalities (14) and (15) we derive the formula for computation of the coefficient $a(\eta)$ of the expansion (9). Substituting the solutions (13) into the corresponding Sohotskii formulas (12), we obtain

$$2\sqrt{\pi}\, i\eta \exp\left(\pm\frac{d}{\eta}\right) a(\eta) = \frac{F^{+}(\eta, \mp d)}{\lambda^{+}(\eta)} - \frac{F^{-}(\eta, \mp d)}{\lambda^{-}(\eta)} . \tag{16}$$

Here

$$F^{\pm}(\eta, -d) = F_1(\eta) \pm i\sqrt{\pi}\, \eta \exp(-\eta^2)\, [H_+(\eta)\, \varphi_1(\eta) + H_+(-\eta)\, C_1] ; \tag{17}$$

$$F^{\pm}(\eta, d) = F_2(\eta) \pm i\sqrt{\pi}\, \eta \exp(-\eta^2)\, [H_+(-\eta)\, \varphi_2(\eta) + H_+(\eta)\, C_2] ; \tag{18}$$

$$F_1(\eta) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} t \exp(-t^2) \left[\frac{C_1}{t+\eta} + \frac{\varphi_1(\eta)}{t-\eta}\right] dt ; \tag{19}$$

$$F_2(\eta) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} t \exp(-t^2) \left[\frac{C_2}{t-\eta} + \frac{\varphi_2(-\eta)}{t+\eta}\right] dt . \tag{20}$$

Substituting first equalities (17) and then (18) into relations (16), we find the formulas for computation of the coefficient $a(\eta)$:

$$\eta \exp\left(\frac{d}{\eta}\right) a(\eta) = \gamma(\eta) \left\{\lambda(\eta)\, [H_+(\eta)\, \varphi_1(\eta) + H_+(-\eta)\, C_1] - F_1(\eta)\right\} , \tag{21}$$

$$\eta \exp\left(-\frac{d}{\eta}\right) a(\eta) = \gamma(\eta) \left\{\lambda(\eta)\, [H_+(-\eta)\, \varphi_2(\eta) + H_+(\eta)\, C_2] - F_2(\eta)\right\} , \tag{22}$$

where

$$\gamma(\eta) = \frac{\eta \exp(-\eta^2)}{\lambda^{+}(\eta)\, \lambda^{-}(\eta)} .$$

Computing integrals of the Cauchy type (19) and (20), we have

$$F_1(\eta) = C_1 t_1(-\eta) + a_1(2-q_1)\, t_2(\eta) + [(1-q_1)\, C_1 - q_1(a_0 - a_1 d + 2U)]\, t_1(\eta) ,$$

$$F_2(\eta) = C_2 t_1(\eta) - a_1(2-q_2)\, t_2(-\eta) + [(1-q_2)\, C_2 - q_2(a_0 + a_1 d - 2U)]\, t_1(-\eta) .$$

Here we obtain

$$t_k(\eta) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{\tau^k \exp(-\tau^2)}{\tau - \eta}\, d\tau , \quad k = 1, 2 ; \quad t_2(z) = \frac{1}{2\sqrt{\pi}} + z t_1(z) .$$

Using the last equalities, we transform formulas (17) and (18). For this purpose, we note that

$$\lambda(\eta)\, [H_+(\eta)\, \varphi_1(\eta) + H_+(-\eta)\, C_1] - F_1(\eta) =$$

$$= q_1 \operatorname{sign}(\eta)(-a_0 - C_1 + a_1 d - 2U)\, t_1(-|\eta|) - a_1(2 - q_1)\, t_2(-|\eta|),$$

$$\lambda(\eta)[H_+(-\eta)\varphi_2(\eta) + H_+(\eta) C_2] - F_2(\eta) =$$

$$= q_2 \operatorname{sign}(\eta)(a_0 + C_2 + a_1 d - 2U)\, t_1(-|\eta|) + a_1(2 - q_2)\, t_2(-|\eta|),$$

and, based on (21) and (22), we obtain the following expression for the continuous-spectrum coefficient:

$$2\eta a(\eta) \sinh\frac{d}{\eta} = \gamma(\eta)\Big\{[-(q_1C_1 + q_2C_2) - a_0(q_1 + q_2) +$$

$$+ (a_1 d - 2U)(q_1 - q_2)] \operatorname{sign}(\eta)\, t_1(-|\eta|) + a_1(q_1 - q_2)\, t_2(-|\eta|)\Big\}.$$

Substituting the coefficients of (14) and (15) into this relation, we find the explicit for the continuous-spectrum coefficient

$$\eta a(\eta) \sinh\frac{d}{\eta} = \frac{Uq_1q_2(q_1 - q_2)}{Q(q_1, q_2)}\gamma(\eta)\left[\operatorname{sign}(\eta)\, t_1(-|\eta|) + \frac{2}{\sqrt{\pi}}\, t_2(-|\eta|)\right]. \tag{23}$$

Below, we will use the value of the integral of expression (23) over the entire real axis:

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \eta a(\eta) \sinh\frac{d}{\eta}\, d\eta = 2U\gamma_1^0 \frac{q_1q_2(q_1 - q_2)}{Q(q_1, q_2)}, \tag{24}$$

$$\gamma_1^0 = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \gamma(\eta)\, t_1(-\eta)\, d\eta = 0.141047.$$

Thus, the boundary-value problem (3) and (7) has completely been solved. In its solution prescribed by equality (5), the function $\psi_c$ is determined by relation (9); the coefficients $a_0$, $a_1$, $C_1$, and $C_2$ are found from equalities (15) and (16), whereas the function $a(\eta)$ is found from (23).

**Macroparameters of the Gas in the Channel (Mass and Heat Fluxes, Friction Force, and Mass Velocity).** We denote the dimensional and dimensionless quantities by unprimed and primed symbols. The mass-flux density will be expressed by the function $\psi$:

$$j_M(x') = \int m v_z f d^3 v = \int m f_0 v_z h d^3 v = \frac{\rho}{2\sqrt{\pi}\beta}\int_{-\infty}^{\infty} \exp(-\mu^2)\, \psi(x', \mu)\, d\mu.$$

Substituting (5) into this equality and using the determination of the gas-mass flux in the direction of the $z$ axis, we write

$$J_M = \frac{1}{\nu\sqrt{\pi}}\int_{-\infty}^{\infty} j_M(x')\, dx' = \frac{\rho}{\nu\beta}\left[a_0 d' + \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} \eta a(\eta) \sinh\frac{d'}{\eta}\, d\eta\right].$$

Replacing here $a_0$ according to (14) and using the value (24) of the integral instead of it, we find that the flux of the gas mass per unit width of the channel is

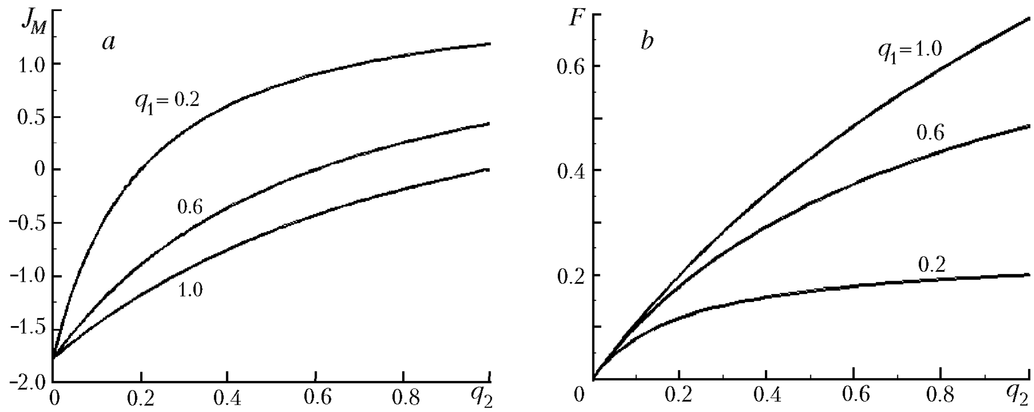



Fig. 1. Mass flux (a) and viscous-friction force (b) vs. $q_2$ for different values of the quantity $q = q_1$; Kn = 1.

$$J_M = \frac{2\rho U' (q_1 - q_2)}{\nu\beta Q (q_1, q_2)} [(\gamma_1^0 - \alpha_0) q_1 q_2 - d'] . \tag{25}$$

We reduce formula (25) to its dimensional form, setting $U' = \sqrt{\beta}\, U$ and $d' = \nu\sqrt{\beta}\, d$. Taking into account that the dynamic viscosity for the BGK model is $\eta = \rho/2\nu\beta$ and selecting the mean free path of molecules $l = \eta\sqrt{\pi}\,\beta/\rho$ according to [5], we find that $d' = \sqrt{\pi}\, d/2l = \sqrt{\pi}/(4\text{Kn})$. Consequently, the mass flux in dimensional form (see Fig. 1a) is

$$J_M = 4\eta \sqrt{\beta}\, U \frac{q_1 - q_2}{Q (q_1, q_2)} \left[\left(\gamma_1^0 - \alpha_0 \frac{\sqrt{\pi}}{4\text{Kn}}\right) q_1 q_2 - \frac{\sqrt{\pi}}{4\text{Kn}}\right]. \tag{26}$$

Here and in what follows we have

$$Q (q_1, q_2) = \left(\frac{1}{2\text{Kn}} - 1 + 4\alpha_0\right) q_1 q_2 + (1 - \alpha_0 q_1 q_2) (q_1 + q_2) .$$

We compute the viscous-friction force in the direction of the $z$ axis per unit area of the surface:

$$F = \int m v_x v_z f d^3 v = \frac{\rho}{2\beta \sqrt{\pi}} \int_{-\infty}^{\infty} \exp (-\mu^2) \mu\psi (x', \mu) d\mu .$$

Substituting (5) into this relation, we find the expression for the friction force in dimensional form (see Fig. 1b):

$$F = -\frac{\rho a_1}{4\beta} = -nkT \frac{a_1}{2} = -2 \sqrt{\frac{\beta}{\pi}}\, pUF_0 (q_1, q_2) . \tag{27}$$

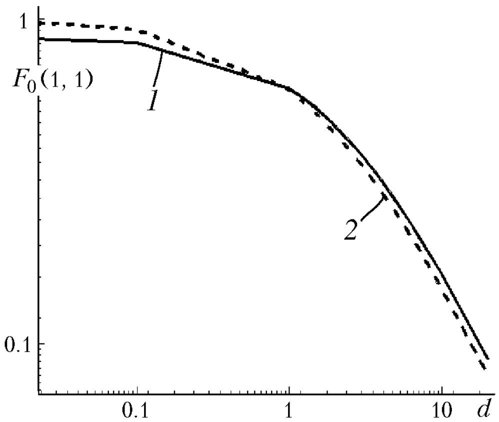
Here

$$F_0 (q_1, q_2) = \frac{q_1 q_2}{Q (q_1, q_2)}$$

is the dimensionless friction factor (see Fig. 2, where a comparison to the existing data [8] is made).

The density of the heat flux in the direction of the $z$ axis is determined by the equality

$$j_Q (x') = \int \frac{m}{2} (v_z - u_z (x')) (\mathbf{v} - u (x'))^2 f d^3 v .$$



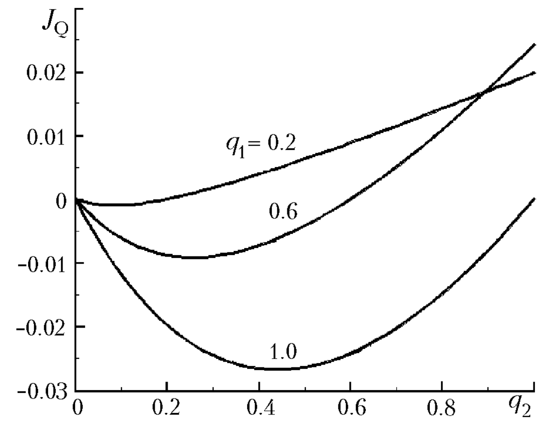




Fig. 2. Friction factor vs. channel width for $q_1 = q_2 = 1$: 1) data from the present work ; 2) [9].

Fig. 3. Heat flux vs. $q_2$ for Kn = 1 and different values of $q_1$.

After the linearization of this expression and passage to the dimensionless molecular velocity, we obtain

$$J_Q = \frac{p}{\sqrt{\pi}} \int_{-d'}^{d'} dx' \int \exp(-C^2)\, C_z \left(C^2 - \frac{5}{2}\right) h\,(x', \mathbf{C})\, d^3C\,.$$

Taking into account that $h = C_z \psi(x, \mu)$, we write

$$J_Q = \frac{p}{2\sqrt{\pi\beta}} \int_{-d'}^{d'} dx' \int_{-\infty}^{\infty} \exp(-\mu^2) \left(\mu^2 - \frac{1}{2}\right) \psi\,(x', \mu)\, d\mu = -\frac{p}{2\sqrt{\pi\beta}} \int_{-\infty}^{\infty} \eta \sinh \frac{d'}{\eta}\, a\,(\eta)\, d\eta\,.$$

Using (24) and passing to dimensionless quantities, we find the expression for the heat flux (see Fig. 3):

$$J_Q = -\gamma_1^0 pU \frac{q_1 q_2 (q_1 - q_2)}{Q\,(q_1, q_2)}\,. \tag{28}$$

The mass velocity of the gas is determined by the second equality from (1), according to which

$$U_z(x') = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-\mu^2)\, \psi\,(x', \mu)\, d\mu\,.$$

After the substitution of the function $\psi(x', \mu)$ into this expression, we obtain that the dimensionless mass velocity is equal to

$$U_z(x') = \frac{1}{2}(a_0 + a_1 x') + \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{x'}{\eta}\right) a\,(\eta)\, d\eta\,. \tag{29}$$

In equality (29), the coefficients $a_0$ and $a_1$ are determined by relations (14), and the function $a(\eta)$ is determined by equality (23). We represent (29) in explicit form (see Fig. 4):

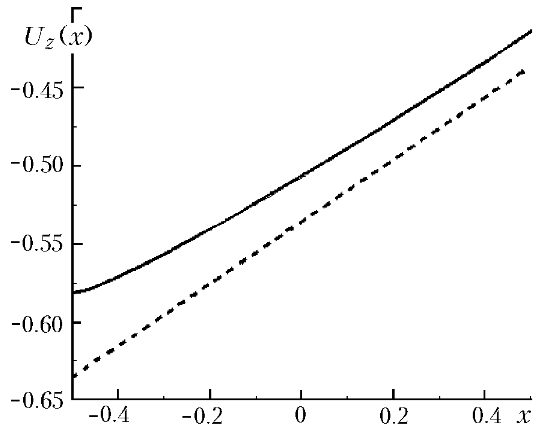



Fig. 4. Mass-velocity profile for $q_1 = 0.75$, $q_2 = 0.25$, Kn = 1, and $2d = 1$.

$$U_z(x) = \frac{U}{Q(q_1, q_2)} \left\{ \frac{q_1 q_2}{\mathrm{Kn}} x + q_1 q_2 (q_1 - q_2)\,[I(x) - \alpha_0] - (q_1 - q_2) \right\},$$

where

$$I(x) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left( -\frac{\sqrt{\pi}\, x}{2\eta \mathrm{Kn}} \right) \frac{b(\eta)\, d\eta}{\sinh(\sqrt{\pi}/4\eta\mathrm{Kn})}; \quad b(\eta) = \frac{\gamma(\eta)}{\eta} \left[ t_1(-|\eta|)\,\mathrm{sign}(\eta) + \frac{2}{\sqrt{\pi}} t_2(-|\eta|) \right].$$

We note that if we set $\alpha_0 = 0$ in the formulas for the macroparameters (26)–(29), the formulas obtained coincide with the corresponding expressions derived in [6] for the case of nearly mirror boundary conditions.

Let us pass to an investigation of the limiting regimes of flow. We consider the case of a wide channel where the Knudsen number is $\mathrm{Kn} = l/2d \ll 1$. There can be two regimes of channel flow of the gas. The first of them corresponds to $\mathrm{Kn} \ll q$, $q = \max(q_1, q_2)$ or, which is the same, $qd/l \gg 1$. It follows from the relations for the macroparameters (26)–(28) that

$$J_M = -2\sqrt{\pi\beta}\, \eta U \frac{q_1 - q_2}{q_1 q_2} (1 + \alpha_0 q_1 q_2), \quad F = -\eta \frac{U}{d}, \quad J_Q = -\frac{\sqrt{\pi}\, \overset{0}{\gamma_1} \eta\, (q_1 - q_2)}{2\sqrt{\beta}\, d}. \tag{30}$$

It is noteworthy that formulas (30) coincide with the corresponding formulas ([6]) derived in the Couette problem with nearly mirror boundary conditions, whereas the relation for the friction force is adequate to the classical formula from [13].

In the second regime of flow of the gas, where $q \ll \mathrm{Kn} \ll 1$, it follows from the relations for the macroparameters that

$$J_M = -2\rho d U \frac{q_1 - q_2}{q_1 + q_2}, \quad F = -\frac{2p\sqrt{\pi/\beta}\, U q_1 q_2}{q_1 + q_2}, \quad J_Q = -\overset{0}{\gamma_1} p U \frac{q_1 - q_2}{q_1 + q_2} q_1 q_2. \tag{31}$$

Expression (31) coincide with the corresponding formulas derived for the case of nearly mirror boundary conditions.

Thus, there is a new regime of flow of the gas, where the expression for the macroparameters is different from the classical ones.

**Conclusions.** We have obtained the solution of the Couette problem in a wide range of Knudsen numbers for arbitrary mirror-diffuse boundary conditions in the case of different tangential momentum accommodation coefficients on channel walls. The distribution function of gas molecules has been constructed in explicit form. It has been shown that the mass and heat fluxes are in proportion to the difference of the accommodation coefficients. The new regime of flow of the gas, which is different from the classical one, has been singled out.

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## NOTATION

$\mathbf{C} = \sqrt{m/2kT}\mathbf{v}$, dimensionless velocity of molecules; $2d$, channel width; $f_0$, absolute Maxwellian; $F$, viscous-friction force; $h$, linear correction to the absolute Maxwellian; $H_+(x)$, Heaviside function, $H_+(x) = 1$, $x > 1$, and $H_+(x)$, $x < 0$; $J_M$, mass flux; $J_Q$, heat flux; $j_M$, mass-flux density; $j_Q$, heat-flux density; $k$, Boltzmann constant; Kn, Knudsen number; $l$, mean free path of molecules; $m$, mass of a gas molecule; $n$, concentration of gas molecules; $\mathbf{n}_1$ and $\mathbf{n}_2$, unit vectors of the normal to the walls in the direction into the channel; $Px^{-1}$, principal value of the integral of $x^{-1}$; $p$, gas pressure; $q_1$ and $q_2$, tangential momentum accommodation coefficients of molecules on the lower and upper plates; $T$, gas temperature; $U_z(x) = \sqrt{\beta}\, u_z(x)$, dimensionless mass velocity in the direction of the $z$ axis; $u_z(x)$, dimensional mass velocity; $\pm U$, velocities of motion of the plates bounding the channel; $\mathbf{v}$, molecular velocity; $\delta(x)$, Dirac delta function; $\eta$, dynamic viscosity; $\lambda(z)$, dispersion function. Subscripts: c, continuous; $M$, mass; $Q$, heat; $z$ and $x$, projections in the direction of the $z$ and $x$ axes.

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